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Interception of Ballistic Targets

by Andrew A. Thompson

ARL-MR-740 April 2010

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Intercepting an incoming artillery or mortar round is receiving interest as an area-protection mechanism. A problem has been that estimation, based on 6-DOF equations for both the incoming round and the interceptor round or missile, is too time consuming from a computational perspective. This report presents a basic simulation in MATLAB that is used to find the maneuver control and the aimpoint or lead angle needed to intercept an incoming round. The method uses reduced dynamics flight models for the interceptor and the incoming round. Also, questions relating to the probability of a hit given a specific standard deviation and bias are addressed.

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Intercepting an incoming artillery or mortar round is receiving interest as an area-protection mechanism. A problem has been that estimation, based on 6-DOF equations for both the incoming round and the interceptor round or missile, is too time consuming from a computational perspective. This report presents a basic simulation in MATLAB that is used to find the maneuver control and the aimpoint or lead angle needed to intercept an incoming round. The method uses reduced dynamics flight models for the interceptor and the incoming round. Also, questions relating to the probability of a hit given a specific standard deviation and bias are addressed.

A trajectory can be adequately modeled by a differential equation. Using the initial conditions, an expected projectile path can be generated. Range and velocity measurements can be used to improve the estimate of the projectile's path. The differential equation and the measurement process are combined to form a Kalman filter. The differential equation models the physics of the trajectory while the measurement updates the parameters of the equation based on least squares estimation. The tradeoffs and dynamics are discussed by Thompson.[1]

2. Simulation

A simulation was devised to indicate the amount of maneuver authority needed to hit an incoming target. The incoming round was modeled using the point mass equations and the interceptor round was modeled with a single dimension differential equation; see Thompson[1] for a discussion. Using these equations, it was possible to compute the lead angle needed for the gun interceptor round. This calculation is based on estimated time of arrival of the interceptor at predicted points of the target trajectory. Figure 1 attempts to illustrate the ideas associated with aimpoint selection. The location of the interceptor is at (5,0). At a given time, the blue line represents the estimated target trajectory moving from the left towards the right. The red points signify points along the trajectory that have the time of arrival of the target also calculated. The green lines represent the interceptor motion to a specific trajectory point. The expected time of the interceptor's arrival at each point can be calculated. At each point, the predicted time until arrival of the target and the interceptor are differenced. An aimpoint/lead angle can be selected for any positive difference by adjusting the firing time of the interceptor so that the time difference is zero. It is also possible to search the interval that goes from negative to positive for

[1]Thompson, A. A. *Ballistic Filtering*; ARL-TR-4735; U.S. Army Research Laboratory: Aberdeen Proving Ground, MD, March 2009.

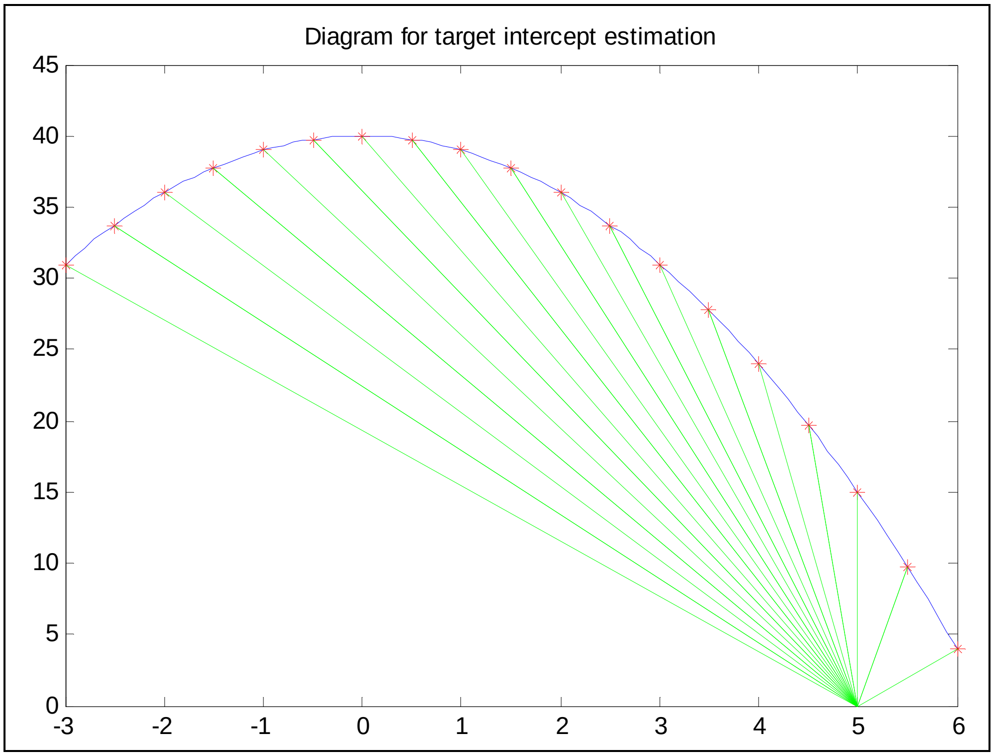

Figure 1. Target-interceptor diagram.

the zero delay aimpoint. The terminal geometry could also be considered in selecting an aimpoint. Post launch adjustments are made by selecting a different predicted target trajectory point for the interceptor to hit. After the interceptor is fired, it could only adjust its course by one degree at a time; thus, corrections could only take place if there was over one degree of error in the predicted impact point.

The accuracy of the target track would have to meet certain criteria before an interceptor could be fired. Meeting this accuracy requirement would ensure that the target would be within the interceptor's maneuver space. Also, in-flight corrections would only be made if the correction statistically exceeds a noise threshold. This latter threshold would keep the interceptor from cycling or correcting and recorrecting the intercept trajectory.

In order to introduce uncertainty in the model, the interceptor speed was changed part way through the flight. In these cases, the initial speed perturbation was followed by several thruster firings. After these firings, the interceptor stayed on an unaltered course until it got near the target. Close to the target several "fine tuning" thruster firings were made. Figure 2 shows a realization of the simulation. In this simulation, the speed of the interceptor was reduced by 30%. The red curve is the target trajectory, the figure shows the entire trajectory as if the target was not intercepted; the blue line is the path of the interceptor; and the green points are the

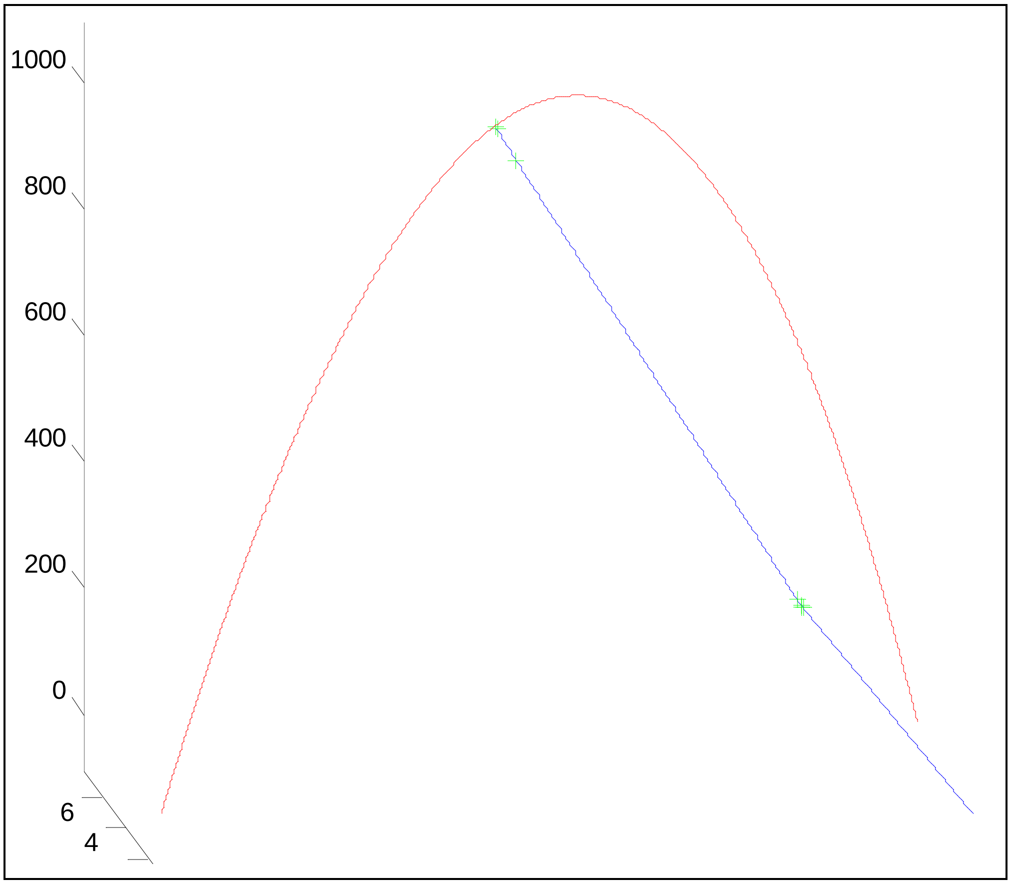

Figure 2. Graphic realization of the simulation.

locations of thruster firings. If one visualizes the predicted target trajectory, increases in the interceptor speed cause the interception point to move back along the trajectory while decreases move the desired aimpoint further forward along the target trajectory. It is obvious that the accuracy of the estimate of the target's trajectory is of basic importance for a system to be successful. Likewise the estimate of the interceptor's trajectory is of equal concern. Thompson[1] discusses these issues. As time increases, the accuracy of both the estimate of the target trajectory and the interceptor trajectory will increase. Many times, polynomial filters are discussed as candidates for trajectory estimation; these filters do not attempt to capture the dynamics of the situation and for longer prediction times are inadequate. Although filters based on 6-DOF models are conceivable, they impose a computational load that cannot be met at this time. Extended Kalman filters using the modified point mass equations present a technique that can be used for this problem. Particle filters also offer a potential method to increase trajectory estimation accuracy over that obtained through recursive least squares polynomial estimation.

3. Probability of Hit

In this section, the statistical issues associated with the probability of hitting a target are discussed using MATLAB as the computational language. There are two families of functions of interest. First, cumulative distribution functions (CDF) give the probability of an event occurring up to a specific input value; since probability can not be negative this function can never decrease as the input value increases. It is often the case that the desired information is to find the value of a distribution associated with a specified cumulative probability. In this latter case, the inverse cumulative distribution function is used.

The root mean square of the target and interceptor estimator errors will result in the overall system error. First, consider a plane perpendicular to the interceptor flight centered on the target, in this plane the position the interceptor strikes is of interest. Consider the target being a circle centered at (0, 0); the probability that the interceptor strikes within this circle is of interest. The first situation to consider is with equal normal errors in both dimensions (X, Y). This question is approached using either a Chi-Square distribution with two degrees of freedom or a Rayleigh distribution. The unit of measure will be a standard deviation unit (sd).

The inverse cumulative distribution function is used to find the values of a distribution associated with specific probabilities. The following MATLAB command is used to find the target radius in standard deviation units for hit probabilities of 0.1 to 0.9 in increments of 0.1 for a Rayleigh distribution. Figure 3 shows the MATLAB syntax and the command output.

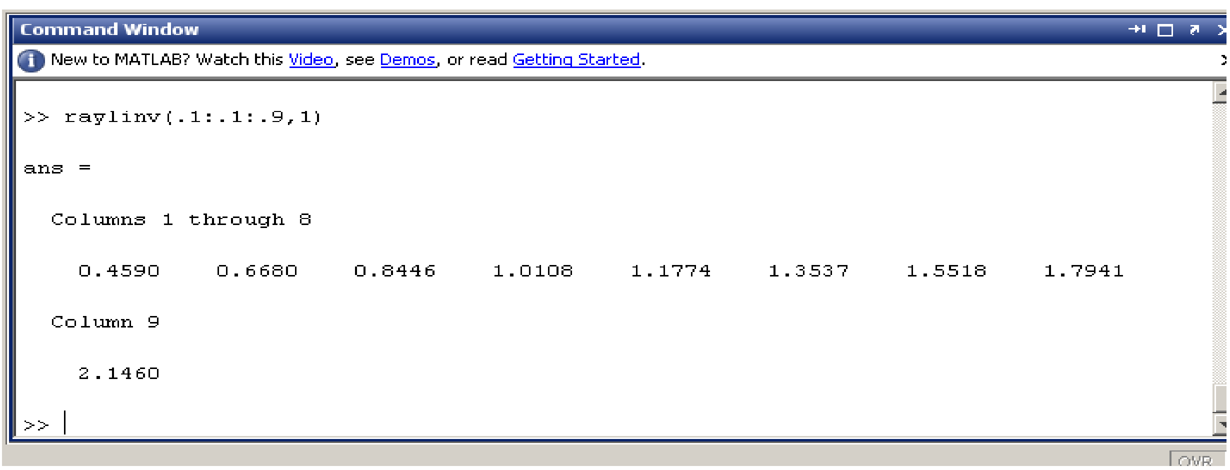

Figure 3. Rayleigh inverse cumulative distribution command.

The following comments show how to interpret the above result. A target with a radius of 0.4590 standard deviation units will have a 0.1 chance of being hit by one round. A target with a radius of 1.1774 standard deviation units will have a 0.5 chance of being hit; this is the value associated with the circular error probable (CEP) for equal errors. Another example would be for standard deviation of 0.1 the target size would have to be 0.21460 to have a probability of 0.9 of being hit.

For a volley of rounds, the probability of missing the target is used to find the probability of at least one round hitting the target. For example, if the standard deviation is 0.2 m and the target radius is 0.2 (0.459) = 0.0918 m, there is a probability of 0.9 of missing the target. A volley of six rounds will have a probability of 0.9^6 or 0.53 of missing the target six times; thus, there is a probability of 0.47 of hitting the target.

The cumulative distribution function returns the probability of an event up to the value given. The next MATLAB command used calculates the cumulative probability of the Rayleigh distribution for targets of radius of 0.2–3 standard deviation units in increments of 0.2 standard deviation units. Figure 4 shows the command and the output generated by the command. The first entry column 1 gives the probability of hitting a target of 0.2 standard deviation units.

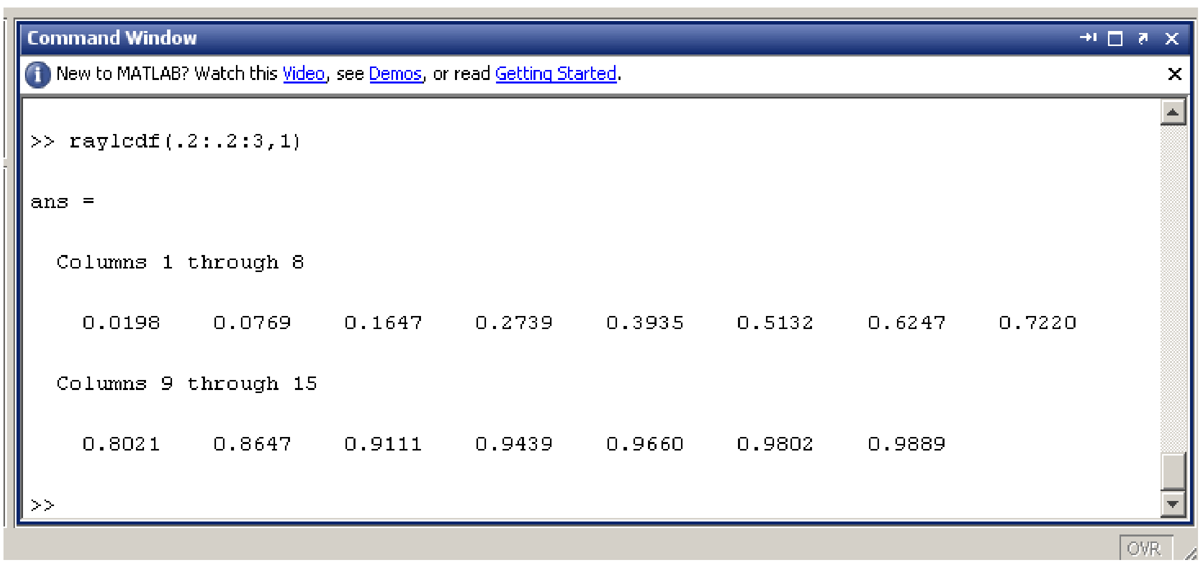

Figure 4. Rayleigh CDF command.

The final entry, column fifteen could indicate that if the standard deviation was 0.1 m there is a probability of 0.9889 of hitting a 0.3-m target.

These ideas can be used to get an idea of the accuracy required to hit a target of a given size. These numbers can be used to generate requirements of outputs of filtering routines and then as requirements for radar data. The issues here are the fidelity of the filter and its prediction ability and then the quality of the data put into the filter. Filters that do a good job of capturing the

dynamics of the situation typically yield lower estimation and prediction errors. Other filters may do well at smoothing but may suffer in prediction. The baseline candidate for prediction and estimation is an extended Kalman filter based on the dynamics of a point mass model. Other models should be compared to these estimators.

This problem can be extended to discuss rectangular targets and targets that are offset from the center (bias); also, the case of different horizontal and vertical errors can be discussed.

4. Extension of Problem

Next, the problem of offset targets will be explored. In this situation, the target is not at the aimpoint this situation can be created by timing errors. Assume the aimpoint is at (0, 0) and the target is at an offset along the x axis. First, rectangular targets will be discussed and then approximations will be made for circular targets. We will assume the standard deviation of the aim error is 1. If this is not the case, just divide all distances by the standard deviation.

Consider a 2×2 rectangle or square target centered on (1, 0). What is the probability of hitting this target when aiming at (0, 0)? The x-axis goes from 0 to 2 and the y-axis goes from –1 to 1. By integrating the normal probability distribution between these limits, a solution is obtained via MATLAB commands. Figure 5 shows the execution of the appropriate commands.

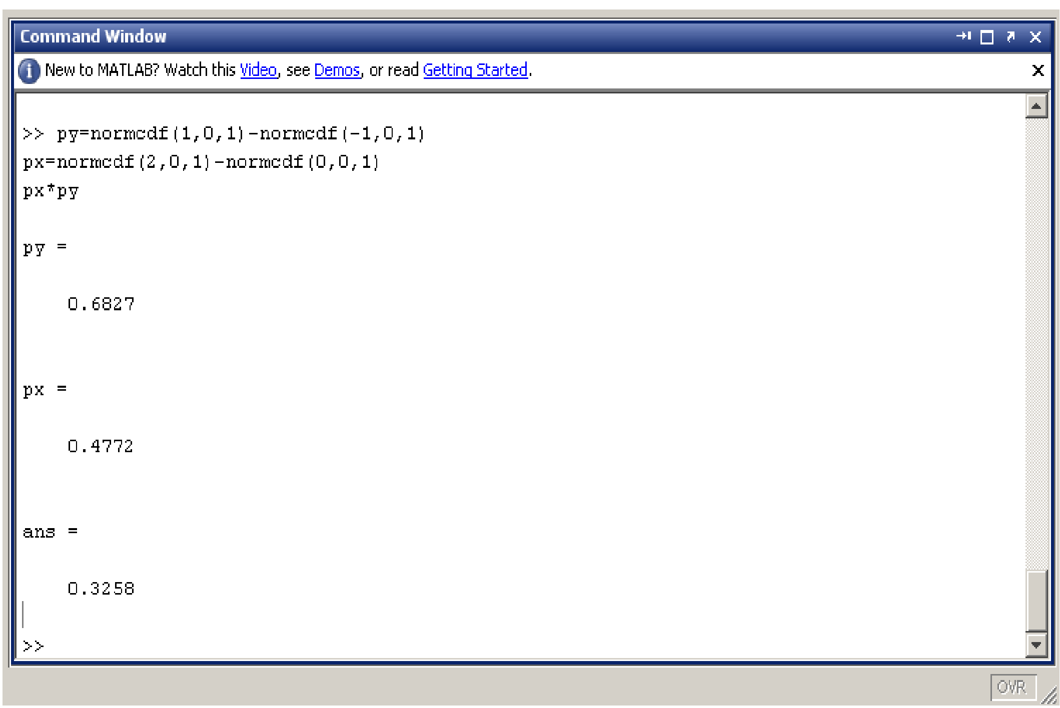

Figure 5. Rectangular target probability calculations.

So, the probability of hitting a square 2 sd square target at an offset of 1 sd unit is 0.3258. If a circular target is desired instead, this result can be multiplied to get an approximation. If the multiplication factor is the ratio of the areas of the inscribed circle to the square, the factor is pi/4. A comparison of the formulas for a rectangle and an ellipse results in the same correction factor.

$$0.3258*pi/4 = 0.2559. \tag{1}$$

So, there is about a 26% chance of hitting a circular target. Consulting DARCOM P 706-101 chapter 14 table 14-2,[2] a precise estimate is found that is 0.2671. The discussion in the DARCOM pamphlet is based on references 3 and 4. The given estimate is precise enough for many applications. The absolute error is 0.01 while the relative error is low at about 4%.

$$(0.2671–0.2559)/0.2671 = 0.0420. \tag{2}$$

If this value is not of sufficient precision, the problem can be solved by using many small rectangles that approximate the target. The Riemann sum can be used to increase the accuracy of the estimate. Using the Riemann sum, a lower and upper probability will be calculated so that the true probability is contained in the resulting closed interval. The Riemann sum is discussed in most elementary calculus books.

Figure 6 shows the preliminary commands used to create the variables needed to calculate the upper and lower probabilities. The first command defines the x values (0 to 2 in steps of 0.01), the second creates the vector of y values that go with the x values (assuming a circle). The offset is represented by x-1. The third line displays the area of interest; this is just the half circle above the x-axis. Next the values of the cumulative distribution are found for each of the x values; and then likewise those for the y values. The y values used are between the x-axis and the calculated y value; thus the calculation will be for a half circle and the result multiplied by two. The final command above finds the probability between consecutive x-values, or the probability associated with the different x intervals. The final steps of the procedure follow. For this problem, we can get an upper and lower Riemann sum as shown in figure 7.

Thus the true probability is between 0.2654 and 0.2686. The average of these is 0.267. As previously mentioned the true value was 0.2671; error is 0.001. This leads to relative errors of

$$(0.2671-0.2654)/0.2671 = 0.0063. \tag{3}$$

The relative error is an order of magnitude less than that of the previous method, similarly

$$(0.2686–0.2671)/0.2671 = 0.0056. \tag{4}$$

[2]DARCOM P 706-101. Chapter 14, table 14-2; U.S. Army Materiel Development and Readiness Command: Alexandria, VA, November 1977.

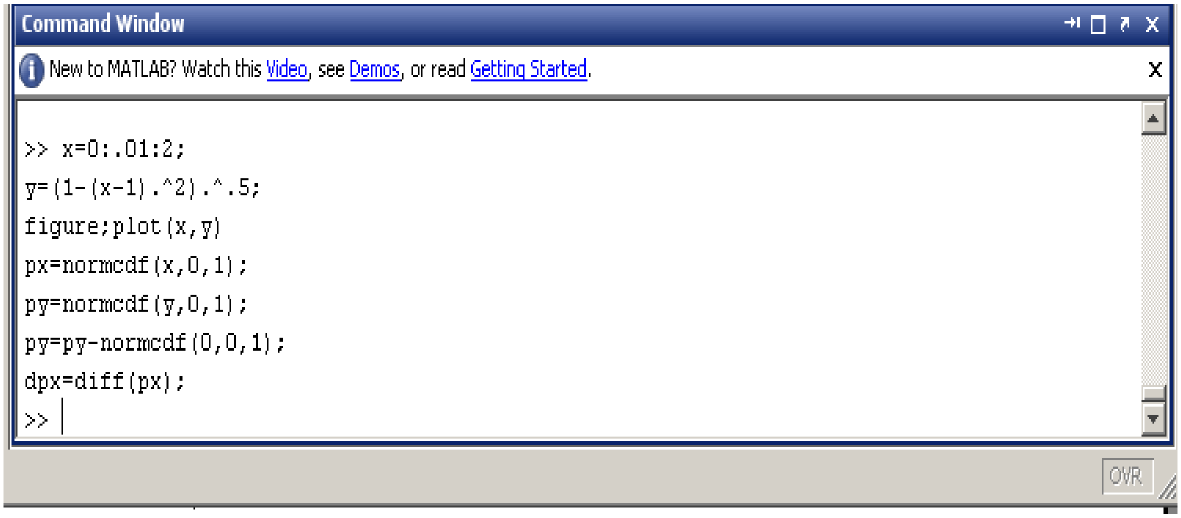

Figure 6. Commands to setup calculation of a Riemann sum.

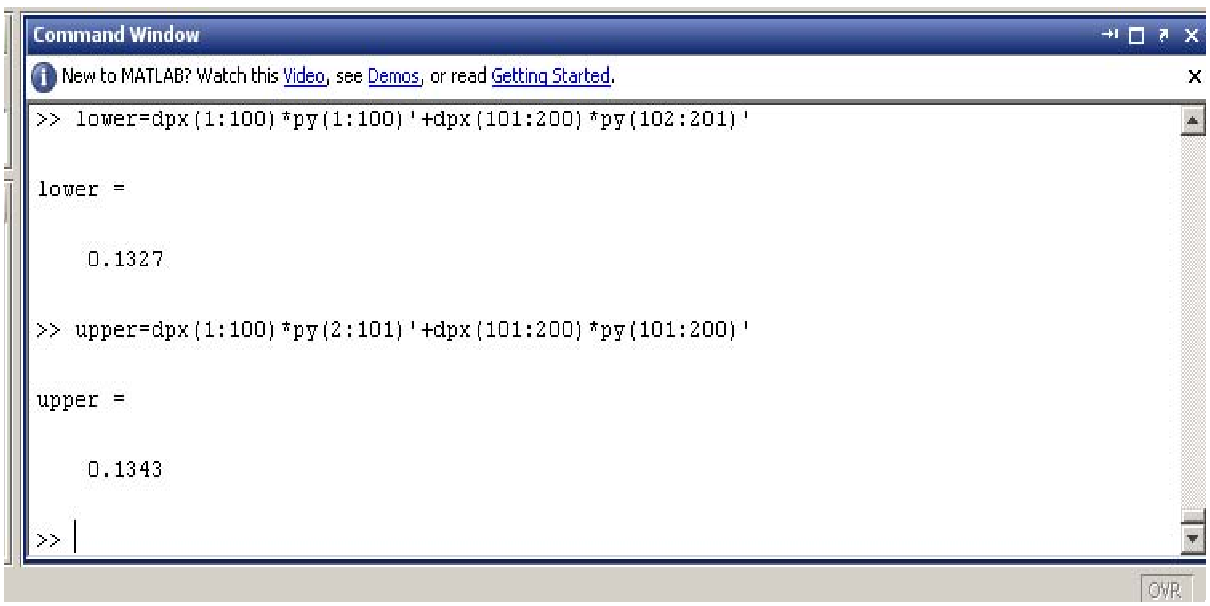

Figure 7. Matlab calculation of upper and lower probabilities.

If the step size is decreased to 0.001, then the corresponding calculations yield a lower sum of 0.2670 and an upper sum of 0.2672 and the error interval is only 0.002. Thus, the procedure given can achieve high accuracy based on the divisions of the x intervals used, or DARCOM P 706-101 can be used. If the divisions of x become extremely small, it is possible that numeric problems could arise; thus, one should gradually shrink the size of the x intervals.

5. Conclusions

Two issues associated with defeating incoming projectiles have been discussed. The aimpoint algorithm is based on timing information associated with the estimators of the target and the interceptor trajectories. Errors in the timing will lead to errors in impact. The simulations indicated two areas of maneuver: first, when errors associated with the trajectories are observed there were firings of 1–4 thrusters; second, as the interceptor closed in on the target there were one to four "fine tuning" thrusters fired. Given there are errors in impact position, methods have been discussed that can be used to quantify performance based on the bias and standard deviation of the impact errors. The performance of the two projectile estimation techniques determines the system error. The methods presented can be used to analyze the ability of an interceptor to defeat an incoming ballistic object.

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